# Laws Governing the Measurable Effects of Pre-Determined Aggregate Percentages to Engineer an Election

Edward Solomon
May 21st, 2021

Abstract:

This paper will demonstrate how to measure the difference between a fair and an unfair election, where an unfair election is an election where the result is predetermined algorithmically.

## Introduction:

Suppose at Lorraine High School, a precinct among many in a particular election, there were two candidates and two methods of voting. The first method of voting would be at the polls on election day; the second mode would be remotely by mail. An unscrupulous actor has already decided that the first candidate will receive exactly 50% of Lorraine High School's vote, regardless of the first candidate's share of the vote on election day. Using a simple equivalence relationship, the malicious actor can adjust the Mail-in percentage in order to achieve a predetermined aggregate result of 50% for the first candidate.

Let us suppose that 1000 persons voted on election day at Lorraine High School, and the first candidate received 750 votes on election day, then the first candidate has 75% of the election day vote at Lorraine.

An additional 1000 persons vote by mail in the Lorraine region; thus a total of 2000 persons voted at Lorraine overall. Since the malicious actor has pre-determined the aggregate percentage to be 50%, then the first candidate will end this election with 1000 votes out of the 2000 total; thus, since the first candidate already has 750 votes, the first candidate will receive an additional 250 votes in the mail, which is 25% of the mail-in vote; such that the combined aggregate, 75% of the election day vote and 25% of the Mail-in Vote results in a 50% Aggregate for the first candidate.

Now let us suppose instead that 2000 persons voted by mail, then the total number of votes at Lorraine would be 3000, and to achieve a 50% aggregate, the first candidate must receive 1500 of those 3000 votes. The first candidate already has 750 votes, and thus they require an additional 750 votes from the mail to sum to 1500. Since 750 divided by 2000 is equal to 37.5%, the first candidate now receives 37.5% of the Mail-in Vote, such that 75% of the Election Day Vote and 37.5% of the Mail-in Vote combines to an aggregate of 50% of the aggregate vote.

We now define a simple parameter, zeta, where $\zeta = \frac{Total\ number\ of\ Mail\ in\ Votes}{Total\ Number\ of\ Election\ Day\ Votes}$, which is the proportion of Mail-in Votes to Election Day Votes; we state the following law that governs the relationship between the Election Day Vote, the Mail-in Vote and the combined Aggregate vote, whether or not the election is fair or unfair:

Let $M = Mail - in\ \ Percentage\ of\ the\ first\ candidate$
Let $E = Election\ Day\ Vote\ \ Percentage\ of\ the\ first\ candidate$
Let $A = Aggregate\ Percentage\ of\ the\ first\ candidate$

$$M = A - \frac{E-A}{\zeta}$$

This Hyperbolic relationship between the modes of voting in respect to a particular candidate forms the foundation of this entire article, for it is this relationship that allows us to measure with absolute certainty whether or not an election was or was not engineered to achieve a predetermined outcome.

As we shall see, if an election is engineered, the above relationship forces a strong positive or negative linear correlation in the activity of the Mail-in Vote in respect to the average value of zeta for a set of precincts; whereas in a fair election, there is no correlation (a flat/zero slope) between the activity of the Mail-in Vote and the value of zeta at each precinct.

# Table of Contents (Add Chap7 Appendices for Deep diving!)

# Chapter 1: Two Candidates, Two Modes

## Section Ⅰ, Initial Definitions

### *Definition 1.1.1; The Precinct Set (Introductory)*

Let **P** be a set of geographically bounded precincts. Let β be the total number of precincts in **P**.
$|\mathbf{P}| = \beta$

### *Definition 1.1.2; The Binary Candidate Set (Introductory)*

Let **C** be the set of candidates, such that $c_1$ is the first candidate and $c_2$ is the second candidate in an election. For this chapter, the first candidate will be named *John* and the second candidate will be named *Fiona*.

Let $|\mathbf{C}| = ψ = 2$

### *Definition 1.1.3; The Binary Mode Set (Introductory)*

Let **M** be the set of election modes, such that $m_1$ is the first mode by which persons may vote; $m_2$ is the second mode by which persons may vote. For this chapter, the first mode shall be the election day vote, and the second mode shall be the Mail-in Vote.

Let $|\mathbf{M}|$ = ना = 2;
Pronunciation of ना / nä /

### *Definition 1.1.4; The Tabulation Tensor (Introductory)*

Let $\mathbf{T}_{\beta,ψ,ना}$ be the Tabulation Tensor, that records the number of votes for each candidate by each mode in each precinct, such that $t_{i,j,k}$ is the vector $(i, j, k)$, and the corresponding value of the cell at the termination of that vector inthe tensor is equal to the number of votes that the candidate, $c_j$, received via that mode, $m_k$, at the precinct, $p_i$.

| i=Precincts j=Candidates | John EDV; $k = 1$ | John Mail-In; $k = 2$ | Fiona EDV; $k = 1$ | Fiona Mail-in; $k = 2$ |
|---|---|---|---|---|
| 1 | 5 | 7 | 2 | 3 |
| 2 | 8 | 6 | 8 | 16 |
| 3 | 4 | 3 | 17 | 11 |
| 4 | 7 | 9 | 4 | 4 |
| 5 | 18 | 21 | 1 | 0 |
| 6 | 12 | 15 | 9 | 3 |
| 7 | 1 | 2 | 11 | 8 |

The cell in yellow is the first candidate's (John's) vote in the second mode (Mail-in) at the fourth precinct, thus: $t_{4,1,2} = 9$.

## *Definition 1.1.5; The Total Matrix (Introductory)*

Let $\mathbf{S}_{\beta,\text{न}}$ be the Total Matrix, that records the number of votes for by each mode in each precinct, such that $s_{i,k}$ is the vector $(i, k)$, and the corresponding value of the cell at the termination of that vector in the matrix is equal to the total number of votes for all candidates via that mode $m_k$ , at the precinct, $p_i$.

$$s_{i,k} = t_{i,1,k} + t_{i,2,k}$$

Based on the previous tables:

| i=Precincts j=Candidates | EDV Total | Mail-in Total |
|---|---|---|
| 1 | 7 | 10 |
| 2 | 16 | 22 |
| 3 | 21 | 14 |
| 4 | 11 | 13 |
| 5 | 19 | 21 |
| 6 | 21 | 18 |
| 7 | 12 | 10 |

## *Definition 1.1.6; The Zeta Column Vector (Introductory)*

Let $\mathbf{Z}_{\beta}$ be the Zeta Vector, that records the proportion of total votes of the second mode against the firstmode at each precinct, such that: $\zeta_i = \frac{s_{i,2}}{s_{i,1}}$.

Based on the previous tables:

| Precinct | Zeta $\zeta(p_i)$ |
|---|---|
| 1 | $10/7 \approx 1.42$ |
| 2 | $22/16 \approx 1.37$ |
| 3 | $14/21 \approx 0.67$ |
| 4 | $13/11 \approx 1.18$ |
| 5 | $21/19 \approx 1.10$ |
| 6 | $18/21 \approx 0.85$ |
| 7 | $10/12 \approx 0.83$ |

## *Definition 1.1.7; The Grand Total Column Vector (Introductory)*

Let $\mathbf{\Omega}_{\beta}$ be the Grand Total Vector, that records the total number of votes at each precinct, such that: $\omega_i = s_{i,1} + s_{i,2}$.

Based on previous tables:

| Precinct | Grand Total |
|---|---|
| 1 | 17 |
| 2 | 38 |
| 3 | 35 |
| 4 | 24 |
| 5 | 40 |
| 6 | 39 |
| 7 | 22 |

## *Definition 1.1.8; The Ratio Tensor (Introductory)*

Let $\mathbf{R}_{\beta,\psi,\text{न}}$ be the Ratio Tensor, that records the number of percentage of the votes received for eachcandidate by each mode in each precinct, such that $r_{i,j,k}$ is the vector $(i, j, k)$, and the corresponding value of the cell at the termination of that vector in the tensor is equal to the share of the vote that the candidate, $c_j$, received via that mode in respect to the total votes of the mode, $s_{i,k}$, at the precinct, $p_i$.

$$r_{i,j,k} = \frac{t_{i,j,k}}{s_{i,k}}$$

Based on Previous Tables:

| i=Precincts j=Candidates | John EDV%; $k = 1$ | John Mail-In%; $k = 2$ | Fiona EDV%; $k = 1$ | Fiona Mail-in%; $k = 2$ |
|---|---|---|---|---|
| 1 | $5/7 \approx 71.42\%$ | $7/10 \approx 70.00\%$ | $2/7 \approx 28.58\%$ | $3/10 \approx 30.00\%$ |
| 2 | $8/16 \approx 50.00\%$ | $6/22 \approx 27.27\%$ | $8/16 \approx 50.00\%$ | $16/22 \approx 72.73\%$ |
| 3 | $4/21 \approx 19.04\%$ | $3/14 \approx 21.42\%$ | $17/21 \approx 80.96\%$ | $11/14 \approx 78.58\%$ |
| 4 | $7/11 \approx 63.63\%$ | $9/13 \approx 69.23\%$ | $4/11 \approx 36.37\%$ | $4/13 \approx 30.77\%$ |
| 5 | $18/19 \approx 94.73\%$ | $21/21 \approx 100.00\%$ | $1/19 \approx 05.27\%$ | $0/21 \approx 00.00\%$ |
| 6 | $12/21 \approx 57.14\%$ | $15/18 \approx 83.33\%$ | $9/21 \approx 42.86\%$ | $3/18 \approx 16.67\%$ |
| 7 | $1/12 \approx 08.33\%$ | $2/10 \approx 20.00\%$ | $11/12 \approx 91.67\%$ | $8/10 \approx 80.00\%$ |

*Definition 1.1.9; The Aggregate Ratio Matrix (Introductory)*

Let $\mathbf{A}_{\beta,\psi}$ be the Aggregate Ratio Matrix, that records the number of percentage of the vote received byeach candidate (across all modes) at each precinct, such that $a_{i,j}$ is the vector $(i, j)$, and the corresponding value of the cell at the termination of that vector in the matrix is equal to the aggregate share of the vote that the candidate, $c_j$, across all modes, in respect to the grand total number of votes recorded at each precinct, $\omega_i$, such that

$$a_{i,j} = \frac{t_{i,j,1}+t_{i,j,2}}{\omega_i}.$$

Based on Previous Tables:

| i=Precincts j=Candidates | John Agg% | Fiona Agg% |
|---|---|---|
| 1 | 12/17 ≈ 70.58% | 5/17 ≈ 29.42% |
| 2 | 14/38 ≈ 36.84% | 24/38 ≈ 63.16% |
| 3 | 7/35 ≈ 20.00% | 28/35 ≈ 80.00% |
| 4 | 16/24 ≈ 66.67% | 8/24 ≈ 33.33% |
| 5 | 39/40 ≈ 97.50% | 1/40 ≈ 02.50% |
| 6 | 27/39 ≈ 69.23% | 12/39 ≈ 30.77% |
| 7 | 3/22 ≈ 13.63% | 19/22 ≈ 86.37% |

*Lemma 1.1.10; Aggregate Lemmas*

Let $\mathbf{A}_{\beta,\psi}$ be the Aggregate Ratio Matrix, that records the number of the percentage of the vote receivedby each candidate (across both modes) at each precinct, such that $a_{i,j}$ is the vector $(i, j)$, and the corresponding value of the cell at the termination of that vector in the matrix is equal to the aggregate share of the vote that the candidate, $c_j$, across both modes, in respect to the grand total number of votes recorded at each precinct, $\omega_i$, such that:

Identity 1 (Absolute Aggregate to Relative Aggregate Identity):

$$a_{i,j} = \frac{t_{i,j,1}+t_{i,j,2}}{\omega_i} = \frac{\left(r_{i,j,1}\right)+\zeta_i(r_{i,j,2})}{\left(\zeta_i+1\right)}$$

Identity 2 (Ratio Identities):

$$r_{i,j,2} = \frac{t_{i,j,2}}{s_{i,2}} = a_{i,j} - \frac{r_{i,j,1} - a_{i,j}}{\zeta_i}; \quad r_{i,j,1} = \frac{t_{i,j,1}}{s_{i,1}} = a_{i,j} + \zeta_i\left(a_{i,j} - r_{i,j,2}\right); \quad \zeta_i = \frac{r_{i,j,1}-a_{i,j}}{a_{i,j}-r_{i,j,2}}$$

Identity 3 (Conservation Identity):

$$1 - a_{i,1} = 100\% - a_{i,1} = a_{i,2}$$

## *Definition 1.1.11; The Reflection Ratio Matrix (Introductory)*

Let $\mathbf{\Lambda}_{\beta,\psi}$ be the Reflection Ratio Matrix, that records the number of the percentage over which both opposing candidate's percentages in opposing modes are reflected over in respect to the proportion between the opposed modes, zeta, at each precinct, such that $\lambda_{i,j}$ is the vector $(i, j)$, and the corresponding value of the cell at the termination of that vector in the matrix is equal to the reflection point of the opposed candidates percentages, $r_{i,1,1}$ and $r_{i,2,2}$, in respect to the proportion of the votes betwixt either mode.

$$\lambda_{i,j} = \frac{t_{i,1,1}+t_{i,2,2}}{\omega_i}.$$

Based on Previous Tables:

| y=Precincts ;x=Candidates | John to Fiona Reflection Percentage | Fiona to John Reflection Percentage |
|---|---|---|
| 1 | 8/17 ≈ 47.05% | 9/17 ≈ 52.95% |
| 2 | 24/38 ≈ 63.15% | 24/38 ≈ 36.85% |
| 3 | 15/35 ≈ 42.85% | 20/35 ≈ 57.15% |
| 4 | 11/24 ≈ 45.83% | 13/24 ≈ 54.17% |
| 5 | 18/40 ≈ 45.00% | 22/40 ≈ 55.00% |
| 6 | 15/39 ≈ 38.46% | 24/39 ≈61.54% |
| 7 | 9/22 ≈ 40.90% | 13/22 ≈ 59.10% |

## *Lemma 1.1.12; Reflection Lemmas*

Let $\mathbf{\Lambda}_{\beta,\psi}$ be the Aggregate Ratio Matrix, that records the number of percentage of the vote received byeach candidate (across all modes) at each precinct, such that $a_{i,j}$ is the vector $(i, j)$, and the corresponding value of the cell at the termination of that vector in the matrix is equal to the aggregate share of the vote that the candidate, $c_j$, across all modes, in respect to the grand total number of votes recorded at each precinct, $\omega_i$, such that

Identity 1 (Abstract Aggregate to Absolute Reflection Identity):

$$\lambda_{i,1} = \frac{t_{i,1,1}+t_{i,2,2}}{\omega_i} = \frac{\left(r_{i,1,1}\right)+\zeta_i\left(r_{i,2,2}\right)}{\left(\zeta_i+1\right)};\ \lambda_{i,2} = \frac{t_{i,2,1}+t_{i,1,2}}{\omega_i} = \frac{\left(r_{i,2,1}\right)+\zeta_i\left(r_{i,1,2}\right)}{\left(\zeta_i+1\right)};\ \lambda_{i,j} = \frac{t_{i,j,1}+t_{i,3-j,2}}{\omega_i} = \frac{\left(r_{i,j,1}\right)+\zeta_i(r_{i,3-j,}}{\left(\zeta_i+1\right)}$$

Observe that the Reflection Point is the abstract aggregate of two opposing candidate's percentages!

Identity 2 (Ratio Identities):

$$r_{i,3-j,2} = \frac{t_{i,3-j,2}}{s_{i,2}} = \lambda_{i,j} - \frac{r_{i,j,1}-\lambda_{i,j}}{\zeta_i};\quad r_{i,j,1} = \frac{t_{i,j,1}}{s_{i,1}} = \lambda_{i,j} + \zeta_i\left(\lambda_{i,j} - r_{i,3-j,2}\right);\quad \zeta_i = \frac{r_{i,j,1}-\lambda_{i,j}}{\lambda_{i,j}-r_{i,3-j,2}}$$

Identity 3 (Conservation Identity):

$$1 - \lambda_{i,1} = 100\% - \lambda_{i,1} = \lambda_{i,2}$$

Identity 4: Opposed Candidate's Second Ratio from the Candidate's Aggregate and Reflection Values

$$(r_{i,\,3-j\,,2}) \;=\; \frac{\lambda_{i,j}\left(\zeta_i+1\right)+\zeta_i-a_{i,1}\left(\zeta_i+1\right)}{2\zeta_i}$$

## *Definitions 1.1.13; The Binary Tabulation Set (Official)*

Let $\mathbf{P}_\beta$ be the set of precincts, such that $|\mathbf{P}_\beta| = \beta$; $\mathbf{P}_\beta = \{p_1,\ p_2 ... p_\beta\}$.

Let $\Gamma_{\tau, n_\tau}$ be the set of registered voters at a precinct, $p_\tau$, such that $\left|\Gamma_{\tau, n_\tau}\right| = n_\tau$; thus the number of voters registered at each precinct is equal to $n_\tau$.

Let $\mathbf{N}_{\tau,k}$ be the singleton set $\{n_{\tau,k}\}$ be a subset of $\Gamma_{\tau, n_m}$; thus, $n_{\tau,k}$ itself is a registered voter.

Then let $\bigcap_{k=1}^{k=n_m} \mathbf{N}_{\tau,k} = \varnothing$. Thus, a voter cannot be registered more than once at a precinct since $\bigcup_{k=1}^{k=n_m} \mathbf{N}_{\tau,k} = \Gamma_{\tau, n_m}$.

Let $\Gamma_\beta$ be the set of all registered voters at all precincts, such that $\bigcup_{i=1}^{i=\beta} \Gamma_{\tau, n_m} = \Gamma_\beta$ and $\bigcap_{i=1}^{i=\beta} \Gamma_{\tau, n_m} = \varnothing$; thus, a voter cannot be registered at more than one precinct.

Let ᚠ be the number of races at a particular precinct. *Pronunciation of* ᚠ */feɪ ħu/*

Let Ψ be the number of candidates for each race.

Let न be the number of modes by which a voter may cast their ballots for a candidate.

Let $\mathbf{R}_{\text{ᚠ}}$ be the set of races common to all precincts in $\mathbf{P}_\beta$.

Let $\mathbf{C}_\psi$ be the set of all candidates for any particular race, $r_k$, such that $f(r_k) = \Psi_k$.

Let $\mathbf{M}_{\text{न}}$ be the set of all modes for which a registered voter may submit their ballot.

Let $\mathbf{W}_{\tau,k,r,c,m}$ be the Binary Tabulation Set that records all tabulations, such that given the legal restriction (where τ is the precinct index, $k$ is the registered voter index, $r$ is the race index, $c$ is the candidate index, $m$ is the mode index):

Then:

$\mathbf{w}_{\tau,k,r,c,m} = 1$ if the registered voter, $n_{\tau,k}$, at precinct $p_\tau$, cast a ballot in race $r$, for candidate $c$, via mode $m$.

*No person (registered voter) who has cast a ballot in a particular race, for any candidate, by any mode, cannot cast any other ballot in the same race, for any candidate (including the candidate they cast their ballot for), by any mode (including the mode by which they cast their ballot); nor, can any registered voter cast a ballot for any race by any mode and then cast another ballot for another race via another mode; thus, the mode in which the registered vote chose to cast their ballot must be uniform for all votes the registered voter cast in all races in which they voted, and they can only vote once in each race.*

*The Binary Tabulation Set can only contain races that are common to all precincts in the Precinct Set; thus, the Presidential Election would be a race common to all precincts, and would therefore exist in the Tabulation Set; however, the local race for the school district's superintendent would only exist for a geographic subset of the precinct set, as such, such a local race cannot be included amongst the set of races for the same Tabulation Set for a set of state or county wide precincts.*

*Any such race where a registered voter can cast a ballot for multiple candidates, by the same mode or multiple modes, must be segregated into a separate universe (tensor). Detecting voter fraud in such a race is beyond the scope of this article*

*Although it would be unusual for the modes between the precincts to be disparate, the Tabulation Set can still persist by recording zeros for all modes that were unavailable to that precinct (empty columns), but available at others. It would be advised to make a declaration of such recordings in order to avoid confusion, as the null data is only being entered to make the Mode Dimension across the precincts uniform. For instance, if one precinct only allowed people to vote by mail, and another precinct only allowed be to vote on Election Day at the poll, then the Election Day Vote at the former precinct would simply be a string of consecutive zeros; likewise, the Mail-in Vote at the latter precinct would also be a string of consecutive zeros.*

Therefore, in accordance to the legal conditions above (one person, one vote, for each race via the same mode):

If $\mathbf{w}_{\tau,\eta,r,c,\delta} = 1$, then… (same mode, δ, for each voter, η, for all races)

$\mathbf{w}_{\tau,k,r,c,m} = 0\ \forall m \leq \text{न},\ m \neq \delta;\ \forall r \leq \text{ᚠ}\ and\ \forall c \leq \Psi_r;\ k = \eta$

If $\mathbf{w}_{\tau,\eta,\Delta,\varepsilon,m} = 1$, then… (maximum of one candidate choice, ε, for each voter, η, for each race, Δ )

$\mathbf{w}_{\tau,k,r,c,m} = 0\ \forall c \leq \Psi_r,\ c \neq \varepsilon; r = \Delta; k = \eta.$

*Bear in mind that the secret ballot statutes in most states in the United States forbid such a tabulation table from existing, such that one cannot extract who any particular voter cast their ballot for, and thus, under the assumption that the law was faithfully executed, that the Precinct Tally Matrix (next definition) for each race would in fact match such a table of tabulations were it to exist.*

## *Example 1.1.14; The Binary Tabulation Set, Two Precincts;*

Let $\beta = 2; n_1 = 7; n_2 = 5$

Let ᚠ = 3; $f(r_1) = 3;\ f(r_2) = 2,\ f(r_3) = 4 \therefore \Psi_1 = 3;\ \Psi_2 = 2;\ \Psi_3 = 4$

Let न = 2

Precinct 1:
Dark Red = Race 1; Dark Blue = Race 2; Neon Green = Race 3;
Yellow = Candidate 1; Orange = Candidate 2; Purple = Candidate 3; Dark Grey = Candidate 4
Light Red = Mode 1; Light Blue = Mode 2

| Race | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Cand | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 1 | 2 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| Mode | 1 | 1 | 1 | 2 | 2 | 2 | 1 | 1 | 2 | 2 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| Alice | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| Beth | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| Cate | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Dan | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Fred | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| Sam | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| Zack | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

Precinct 2:

| Race | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Cand | | | | | | | | | | | | | | | | | | |
| Mode | | | | | | | | | | | | | | | | | | |
| Hal | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| Ish | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| Jake | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| Max | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| Nan | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |

## *Definitions 1.1.15; The Precinct Tally Matrix (Official)*

Let $v(k)$be the function of some race, *k*, that generates the *Precinct Tally Matrix* **V,*k***$_{[\beta, भ]}$ of dimensions β by भ, where $भ = \Psi न$.
$\Psi = f(r_k)$ ; *Pronunciation of* भ */bə̤/*

*This matrix utilizes the zero index for both the candidate number and the mode number for computations.*

For some element,$v_{\tau, j}$, of the matrix **V,*k***$_{[\beta, भ]}$, let the floor function, $\delta = \lfloor \frac{j}{न} \rfloor$return the mode of the casted ballot and let $\varepsilon \equiv j \bmod \Psi$ return the candidate chosen; $j = \varepsilon + \Psi\delta$; $j \in \mathbb{Z}$; $\chi = \varepsilon + i\delta$; $\chi \in \mathbb{C}$

Then let $$v_{\tau,\chi} = v_{\tau,(\varepsilon+i\delta)} = v_{\tau, j} = \sum_{k=1}^{k=n_\tau} w_{\tau,k,r,\,\delta,\,\varepsilon} .$$

Let भ be the Cartesian Product of $\mathbf{C}_\psi$ and $\mathbf{M}_न$, for a particular race, *k*. This shall be known as the set; | |=भ .

If the votes are tallied by hand on sheet then the unary sum $\sum_{k=1}^{k=n_\tau} w_{\tau,k,r,\,\delta,\,\varepsilon}$ must be converted to a non-unary numeral system for each cell of the Precinct Tally Matrix. This article assumes that the decimal system is used and that all votes are integers in a fair election.

Although not required for a fair election, the complex number χ is a powerful tool for investigating and detecting fraud (in particular for the detection of *Reflection Fraud*), since each row vector, τ, of the matrix $\mathbf{V}_{\beta\text{-}1, भ\text{-}1}$, is a one-dimensional representation (*vectorization*) of an isolate tabulation matrix of length Ψ (candidates) and height न (modes) for each precinct for a single race.

*Vectorization*:
https://link.springer.com/chapter/10.1007%2F978-1-4757-3238-2_4
Vectorization (mathematics)

Spreadsheets can convert these one-dimensional representations into a matrix using the Pivot Table option (also known as *Matricization*, which is the simplest form of multilinear tensor reshaping, namely, a one-dimensional tensor to a two-dimensional tensor):
ArrayReshape—Wolfram Language Documentation
Tensor Decompositions and Applications

It is not uncommon for county precinct data (the tally matrix) to be stored in spreadsheet applications via cyclic set of embedded pivot tables for each precinct (a block matrix); nor it is difficult to undo such a spreadsheet format by assigning all row numbers a residue of a modulus equal to the height of eachpivot table (each block) and executing a formula across a range of columns equal to the area of the individual pivot tables (area of the blocks), as such, the spreadsheet transformation is the consecutive linearization of the embedded pivot tables (blocks) themselves.

Ultimately, detecting algorithmic election fraud involves massive parallel computation of vectorized tallies to reduce the multilinear complexity of all the combinations of candidates and modes (a power set of size $2^{\Psi+न}$) into a more simple computation of only linear complexity. The usage of complex χ against the vectorized tallies allows us to execute an organized and systematic approach to revealing any fraud that may have occurred.

Multilinear algebra - *"In a vector space of dimension n, one usually considers only the vectors. According to Hermann Grassmann and others, this presumption misses the complexity of considering the structures of pairs, triples, and general multivectors. Since there are several combinatorial possibilities, the space of multivectors turns out to have $2^n$ dimensions."*

## *Example 1.1.16; Precinct Tally Matrix, Two Precincts.*

*Precincts 1 and 2*; **Race 3**: (from above example):

**V,*k***$_{[2,8]}$

| *j* | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| τ = 1 | 0 | 0 | 2 | 0 | 0 | 1 | 1 | 1 |
| τ = 2 | 0 | 0 | 1 | 0 | 0 | 2 | 2 | 0 |

**V,*k***$_{[2,8]}$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| χ = ε + | 0,0*i* | 1,0*i* | 2,0*i* | 3,0*i* | 0,1*i* | 1,1*i* | 2,1*i* | 3,1*i* |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\tau = 1$ | 0 | 0 | 2 | 0 | 0 | 1 | 1 | 1 |
| $\tau = 2$ | 0 | 0 | 1 | 0 | 0 | 2 | 2 | 0 |

## *Definitions 1.1.17; The Tally Matrix Subset and Combinatorial Definitions for some race, k.*

### *Universe Sets*

Candidate Universe Set, for any race, *k*, the set of all candidates in that race shall be $\mathbf{C}_{\psi}$.
Mode Universe Set, for any race, *k*, the set of all modes in that race shall be $\mathbf{M}_{\text{न}}$.
Universe Set, for any race, *k*, the cartesian product $\mathbf{C}_{\psi}$ of $\mathbf{M}_{\text{न}}$ and shall be $_{\text{भ}}$.
Precinct Universe Set, for any race, *k*, the set of all modes in that race shall be $\mathbf{P}_{\beta}$.

$|\mathbf{C}| = \psi$ ; $|\mathbf{M}| = \text{न}$ ; $|\ \ | = \text{भ}$ ; $|\mathbf{P}| = \beta$

### *Recognition and Discard Sets,* $\mathbf{S}'$ *and* $\neg\mathbf{S}'$.

For any particular race, *k*, let $\mathbf{C}' \subset \mathbf{C}_{\psi}$ and let $\neg\mathbf{C}' = \mathbf{C}_{\psi} - \mathbf{C}'$.
For any particular race, *k*, let $\mathbf{M}' \subset \mathbf{M}_{\text{न}}$ and let $\neg\mathbf{M}' = \mathbf{M}_{\text{न}} - \mathbf{M}'$.
For any particular race, *k*, the cartesian product $\mathbf{C}'$ of $\mathbf{M}'$ and shall be '.
For any particular race, *k*, let $\mathbf{P}' \subset \mathbf{P}_{\beta}$ and let $\neg\mathbf{P}' = \mathbf{P}_{\beta} - \mathbf{P}'$.

$|\mathbf{C}| = \psi_2$ ; $|\mathbf{M}| = \text{न}_2$ $|\ '| = \text{भ}_2$ $|\mathbf{P}| = \beta_2$;

These are the candidate, mode and precincts recognition and discard subsets. Isolating portions of the universe into these smaller subsets allows for faster computation.

Unfortunately, there is no method that allows spreadsheet users to utilize the *Recognition*, *Discard*, *Hybrid* and *Dormant* Sets; thus, spreadsheet users will immediately proceed to the *Partition*n, *Focus* and *Contrast* Sets. This article will provide the framework and implementation for both script writers (coders) and spreadsheets.

**It is imperative that the spreadsheet implementation also be described, since spreadsheets are the only tool that ordinary citizens have access to and knowledge of.**

### *The Hybrid and Dormant Sets,* $\mathbf{S}''$ *and* $\neg\mathbf{S}''$.

For any particular race, *k*, let $\mathbf{C}' \subset \mathbf{C}_{\psi}$ and let $\neg\mathbf{C}' = \mathbf{C}_{\psi} - \mathbf{C}'$.
For any particular race, *k*, let $\mathbf{M}' \subset \mathbf{M}_{\text{न}}$ and let $\neg\mathbf{M}' = \mathbf{M}_{\text{न}} - \mathbf{M}'$.
For any particular race, *k*, let $\mathbf{P}' \subset \mathbf{P}_{\beta}$ and let $\neg\mathbf{P}' = \mathbf{P}_{\beta} - \mathbf{P}'$.

et $_{\text{भ}}$ be the Cartesian Product of $\mathbf{C}_{\psi}$ and $\mathbf{M}_{\text{न}}$, for a particular race, *k*. This shall be known as the set; $|\ \ | = \text{भ}$ .

Candidate Focus Set: $\mathbf{C}''$; Anti-Candidate Focus Set: $\neg\mathbf{C}''$.
For any particular race, *k*, let $\mathbf{C}'' \subset \mathbf{C}'$ and let $\neg\mathbf{C}'' = \mathbf{C}' - \mathbf{C}''$.

Mode Recognition Set: $\mathbf{M}'$; Mode Discard Set: $\neg\mathbf{M}'$
For any particular race, *k*, let $\mathbf{M}' \subset \mathbf{M}_{\text{न}}$ and let $\neg\mathbf{M}' = \mathbf{M}_{\text{न}} - \mathbf{M}'$.

Mode Focus Set: $\mathbf{M}''$; Anti-Mode Focus Set: $\neg\mathbf{M}''$.
For any particular race, *k*, let $\mathbf{M}'' \subset \mathbf{M}'$ and let $\neg\mathbf{M}'' = \mathbf{M}' - \mathbf{M}''$.Definition 1.1.25; The Hybrid and Dormant Sets

Let Hभ 3 be a subset of Bभ 2; Hभ 3 = **B**भ2,2; this is the Hybrid Set. |Hभ 3| = भ3.

Let ¬H¬ऋ3 = Bऋ2 - Hऋ3 be the Dormant Set; ¬H¬ऋ3 = **B**ਮ2,⌐2. = **B**ਮ2,2ਮ2-1 - 2 ; |¬H¬ਮ 3 | = ¬ਮ3.
Definition 1.1.26; The Partition and Anti-Partition Sets.

Let SЖ be a subset of Hऋ3; SЖ = **H**ਮ3,3; this is the Partition Set. |SЖ| = Ж .
Let ¬S¬Ж = Hऋ3 - SЖ be the Anti-Partition Set; ¬S¬Ж = **H**ਮ3,¬3. = **H**ਮ3,2ਮ3-1 - 3. |¬S¬Ж| = ¬Ж .
Definition 1.1.27; The Focus and Anti-Focus Sets.

Let FΛъ be a subset of SЖ; FΛъ = **S**Ж,4; this is the Focus Set. |FΛъ| = Λъ .
Let ¬F¬Λъ = SЖ - FΛъ be the Anti-Focus Set; ¬F¬Λъ = **S**Ж,¬4 = **S**Ж,2Ж-1 - 4 . |¬F¬Λъ| = ¬Λъ .
Definition 1.1.28; The Contrast and Anti-Contrast Sets.

Let GЯ be a subset of ¬SЖ; GЯ = ¬**S**Ж,5; this is the Contrast Set. |GЯ | = Я .
Let ¬G¬Я = ¬SЖ - GЯ be the Anti-Contrast Set; ¬G¬Я = ¬**S**Ж,¬5= **S**Ж,2Ж-1 - 5 . |¬G¬Я| = ¬Я .
Definition 1.1.29; Partition Zeta

Let ζЖ be proportion of all votes in the Anti-Partition to the Partition.

Ж=j=1j= ¬Ж¬sjk=1k= Жsk

Definition 1.1.30; Focus and Anti-Focus Percentage

Focus Percentage: fΛъ=t=1t= Λъftk=1k= Жsk Anti-Focus Percentage: ¬f¬Λъ=t=1t= Λъ¬ftk=1k= Жsk 1-fΛъ=¬f¬Λъ.

Definition 1.1.31; Contrast and Anti-Contrast Percentage

Contrast Percentage: gЯ=w=1w= Яgwj=1j= ¬Ж¬sj Anti-Contrast Percentage: ¬g⌐Я=w=1w= Я¬gwj=1j= ¬Ж¬sj
1-gЯ=¬g¬Я.

Definition 1.1.32; Focus Aggregate and Focus Reflection(Focus to Contrast)

Focus Aggregate: Я=fΛъ+ЖgЯЖ+1 Focus Reflection: Я=fΛъ+Ж⌐g⌐ЯЖ+1 1-Я=Я.

Definition 1.1.33; The Focus Aggregate and Reflection to Anti-Contrast Percentage Conversion Identity.

## *Definition 1.1.18a; Binary Mask Indexing*

Let the elements in **S**,|**S**|=$n$, be assigned a number by their index,κ, and let that number be the function$\xi(n, \kappa) = 2^{n-\kappa}$.
*In this article we shall always start with the zero index unless specified otherwise.*

Then let $\xi_n$ be a row vector of length $n$, such that $\xi_\kappa = 2^{n-\kappa}$ $\forall \kappa \leq n$

$n = 8$

| **S** | $s_0$ | $s_1$ | $s_2$ | $s_3$ | $s_4$ | $s_5$ | $s_6$ | $s_7$ |
|---|---|---|---|---|---|---|---|---|
| $\xi_n$ | 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 |

Although one may rewrite $\xi(n, \kappa) = 2^{n-\kappa}$ as $\xi(n, \kappa) = 2^{\kappa}$, numeral systems are most often read from left-to-right by most cultures; thus it was decided that $\xi(n, \kappa) = 2^{n-\kappa}$ would be the implementation of the function since binary numerals are also read from left-to-right.

## *Definition 1.1.18b; Boolean Superset Function*

Let **S** ⊃ **Q**; |**S**| = $n$.

Then let ᚦ[**S**,**Q**] be a column vector ᚦ$_{SQ}$ of length *n* such that ᚦ$_\kappa = 1 \leftrightarrow s_\kappa \in$ **Q**; ᚦ$_\kappa = 0 \leftrightarrow s_\kappa \notin$ **Q**

**Q** = $\{s_2, s_3, s_4, s_6, s_7\}$; |**S**| = *8*

| **S** | $s_0$ | $s_1$ | $s_2$ | $s_3$ | $s_4$ | $s_5$ | $s_6$ | $s_7$ |
|---|---|---|---|---|---|---|---|---|
| **Q** | | | $q_0 = s_2$ | $q_1 = s_3$ | $q_2 = d_4$ | | $q_3 = s_6$ | $q_4 = s_7$ |
| ᚦ$_{SQ}$ | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 |

## *Definition 1.1.18c; Hadamard Transformation and the Subset Index and the Complement Subset Index Theorem*

Let **S** ⊃ **Q**; |**S**| = *n*.

Let the vector ᛉ be the Hadamard Product of $\xi_n$ and the transposition of ᚦ$_{SQ}$ , ᛉ$_{SQ} = \xi \circ ($ᚦ$_{SQ})^T$

*Pronunciation of* ᛉ / æl gi:z /

ᛉ $= \xi \circ ($ᚦ$_{SQ})^T$

| $\xi_n$ | 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 |
|---|---|---|---|---|---|---|---|---|
| ᚦ$_{SQ}$ | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 |
| ᛉ$_{SQ}$ | 0 | 0 | 32 | 16 | 8 | 0 | 2 | 1 |

Then let $\Delta = \sum_{j=1}^{j=n}$ ᛉ$_j$ be known the ***Subset Index,*** such that **Q** = **S**$_\Delta$ is an explicit subset of **S**. $\Delta = 59$ in the above example.

Then if **F** = **S** - **Q**; **Q**= **S**$_\Delta$ , |**S**| = *n*; then **F** = **S**$_{(2^n-1)-\Delta}$.

***Proof***:

Let $g($ᚦ$_{SQ})$ transform the vector ᚦ$_{SQ}$ to the vector ¬ᚦ$_{SQ}$ such that ¬ᚦ$_{SD,j} = (1 -$ ᚦ$_{SQ,j}),\ \forall j \le n$ .

Then the vector subtraction of ᚦ$_{SS}$(which would be a string of *n* consecutive 1's) and ¬ᚦ$_{SQ}$ is the same as ᚦ$_{SF}$.

*From above*: $\neg\Delta = 196;\ 59 + 196 = 255 = 2^8 - 1$ Q.E.D.

| ᚦ$_{S(S-Q)}$ | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|

## *Definition 1.1.19; The Tally Recognition Vector*

Let $ᚦ_{भ}$ be the *Tally Recognition Vector* in respect to $_{भ}$, such that $ᚦ_{भ} = vec(ᚦ_{MM'}ᚦ_{CC'})^{\mathbf{T}} = (ᚦ_{CC'} \otimes \mathbf{I}_{न})(ᚦ_{MM'})^{\mathbf{T}}$ ; $भ = Ψन$

Suppose that we suspected that algorithmic fraud was executed only between a subset of the candidates and a subset of the modes betwixt them, and that the remaining the candidates and all of their respective modes were not affected, then how could transform the Precinct Tally Matrix dynamically to investigate this suspicion, in a manner that allows us to rapidly examine all possible combinations of those subsets of candidates and modes.

Let us consider the case of six candidates and four modes. We suspect that tallies of Candidates 0, 1 and 4 in modes 1 and 3 are the tallies that were manipulated, then we proceed the following manner.

Let the Candidate Recognition Set $\mathbf{C'} = \{c_0, c_1, c_4\}$, $\Delta = 50$; $\mathbf{C'} = \mathbf{C}_{50}$.

| **C** | $c_0$ | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|---|---|---|---|---|---|---|
| $\xi_6$ | 32 | 16 | 8 | 4 | 2 | 1 |
| $ᚦ_{\mathbf{CC'}}$ | 1 | 1 | 0 | 0 | 1 | 0 |
| $ᛉ_{\mathbf{CC'}}$ | 32 | 16 | 0 | 0 | 2 | 0 |

Let the Mode Recognition Set $\mathbf{M'} = \{m_1, c_3\}$, $\Delta = 5$; $\mathbf{M'} = \mathbf{M}_5$.

| **M** | $m_0$ | $m_1$ | $m_2$ | $m_3$ |
|---|---|---|---|---|
| $\xi_4$ | 8 | 4 | 2 | 1 |
| $ᚦ_{\mathbf{MM'}}$ | 0 | 1 | 0 | 1 |
| $ᛉ_{\mathbf{MM'}}$ | 0 | 4 | 0 | 1 |

We now vectorize the product of the vectors $vec((ᚦ_{MM'})(ᚦ_{CC'}))$, where $(ᚦ_{MM'})$ is the column vector and $(ᚦ_{CC'})$ is the row vector.

To perform this operation we consider both vectors as matrices, such that $ᚦ_{CC'}$ is a Ψ by 1 matrix (single row matrix) and $ᚦ_{MM'}$ is a 1 by न matrix (single column matrix) and use the following formula to linear this product:

$vec(\mathbf{XY}) = (\mathbf{I_m} \otimes \mathbf{X})vec(\mathbf{Y}) = vec(\mathbf{XY}) = (\mathbf{Y}^{\mathbf{T}} \otimes \mathbf{I_k})vec(\mathbf{X})$ . Thus, let $(ᚦ_{MM'}) = \mathbf{X}$ and let $(ᚦ_{CC'})$ be equal to $\mathbf{Y}$ $(ᚦ_{CC'})$; $k =$ न ; $m = Ψ$.

Where $\mathbf{X}$ is a $k$ by $l$ matrix and $\mathbf{Y}$ is a $l$ by $m$ matrix, $\otimes$ is the Kronecker Product and $\mathbf{I_m}$ is the square $m$ by $m$ identity matrix.
["Kronecker Product." From MathWorld - A Wolfram Web Resource.](#)
[Wikipedia on the Kronecker product](#)

Since both $\mathbf{X}$ and $\mathbf{Y}$ are both single row and column matrices, respectively, they are already vectorized, allowing us to simply take the Kronecker Product of an identity matrix and a vector, all values of which would be zero (besides the repeating diagonal terrace of the one-dimensional block matrix). We would then multiply this trivial array by column vector $\mathbf{Y}$, since $\mathbf{Y}$ by its own nature was already vectorized! We need not vectorize that which is already vector!

The following tool vectorizes the resultant matrix of a column vector (red) and a row vector (blue), the vector lengths are manually input in the yellow cells B4 and F1 (length must be greater than 1). Row 2 of the sheet contains the vectorized form of the resultant matrix.

Vectorizer: 2DMatrix -Kronekcer Method; 12x12 maximum

From the above tool, we only allow the vector cell entries to be 0 or 1, producing $vec((ᚦ_{MM'})(ᚦ_{CC'}))$.

Vectorizer: 2DMatrix; Tabulation ᚦ (thorn) Product; 12 candidates and 12 modes maximum

We then transpose the resultant column vector from this operation (already done by the spreadsheet tool) into the desired row vector.

**The spreadsheet application runs the following algorithm** (in red on the right-hand-side).
$vec(\mathbf{XY})^{\mathbf{T}} = ((\mathbf{Y}^{\mathbf{T}} \otimes \mathbf{I_k})vec(\mathbf{X}))^{\mathbf{T}} = ((\mathbf{Y}^{\mathbf{T}})^{\mathbf{T}} \otimes (\mathbf{I_k})^{\mathbf{T}})vec(\mathbf{X})^{\mathbf{T}} = (\mathbf{Y} \otimes \mathbf{I_k})vec(\mathbf{X})^{\mathbf{T}} = (ᚦ_{CC'} \otimes \mathbf{I}_{न})vec(ᚦ_{MM'})^{\mathbf{T}}$.

Since $ᚦ_{MM}$ is already a vector this reduces to: $vec(ᚦ_{MM'}ᚦ_{CC'})^{\mathbf{T}} = (ᚦ_{CC'} \otimes \mathbf{I}_{न})(ᚦ_{MM'})^{\mathbf{T}}$.

## *Definition 1.1.20; The Governatrice Matrix, Constant and Set*

Let ✡ be a column vector of length **β**, such that $✡_\tau = 1\ \forall \tau \le \beta$

Let ♀,*k*$_{[\beta,\text{म}]}$ be a **β** x म matrix that is the product of the column vector ✡ and the row vector ᚦ$_{\text{अ}}$, for a particular race, *k*.

Then ♀,*k*$_{[\beta,\text{म}]}$ matrix shall be known as the *Governatrice Matrix* for race *k*.

Let $\text{अ}_2 = \sum_{j=0}^{j=\beta-1} ᚦ_{\text{अ},j}$, this is *Governatrice Constant*; let $\neg\text{अ}_2 = \text{अ} - \text{अ}_2$ be the *Discarded* **χ** *Constant*.

Let $\mathbf{B}_{\text{अ}\,2}$ be the subset of $_{\text{अ}}$; $\mathbf{B}_{\text{अ}\,2} = {}_{\text{अ},\Delta}$; $|\mathbf{B}_{\text{अ}\,2}| = \text{अ}_2$.
Let $\neg\mathbf{B}_{\neg\text{अ}\,2} = {}_{\text{अ}} - \mathbf{B}_{\text{अ}\,2}$; $\neg\mathbf{B}_{\neg\text{अ}\,2} = {}_{\text{अ},\neg\Delta} = ?_{\text{अ},(2^{\text{अ}}-1)-\Delta}$; $|\neg\mathbf{B}_{\neg\text{अ}\,2}| = \neg\text{अ}_2$.

## *Definition 1.1.21; The Recognized Tally Matrix*

Let the Hadamard Product (element-wise product) of ♀,*k*$_{[\beta,\text{म}]}$ and **V,*k***$_{[\beta,\text{म}]}$ be the *Recognized Tally Matrix* **A,*k***$_{[\beta,\text{म}]}$ .

[Linear Algebra — ML Glossary documentation]
[Hadamard product (matrices)]

## *Definition 1.1.22; The Precinct Recognition Vector and the Criteria Set*

Let the column vector ᛟ$_{\beta,z}$ be the *Precinct Recognition Vector*, such that ᛟ$_{\tau,z} = 0 \text{ or } 1\ \forall\tau \le \beta$. ᛟ: othala; IPA: o:θ-a:-l a:

The evaluation of each cell is determined by an arbitrary boolean function. For instance, if we wanted to only consider the subset of precincts such that the Election Day vote percentage of one candidate was greater than the Election Day vote percentage of another candidate, then we would evaluate each cell of ᛟ$_\beta$ by that criteria.

For this reason, we shall invoke the necessity of a *Criteria Set* of functions, **S**, $|\mathbf{S}| = s$, where $s_z$ indicates a particular criterion.

## *Definition 1.1.23; The Governatore Matrix and Constant*

Let ☩ be a row vector of length अ, such that ☩$_j = 1\ \forall j \le \text{अ}$

Let ♂,*k*$_{[\beta,\text{म}]}$ be a **β** x म matrix that is the product of the column vector ᛟ$_{\beta,z}$ and the row vector ☩, for a particular race, *k*.

Then ♂,*k*$_{[\beta,\text{म}]}$ matrix shall be known as the *Governatore Matrix* for race *k*.

Let $\beta_2 = \sum_{j=0}^{j=\beta-1} ᚦ_{\text{अ},j}$, this is *Governatore Constant*; let $\neg\beta_2 = \beta - \beta_2$ be the *Discarded Precinct Constant*.

## *Definition 1.1.24a; The Recognized Precinct-Tally Matrix*

Let the Hadamard Product (element-wise product) of ♂,$k_{[\beta, \text{भ}]}$ and **A,**$\boldsymbol{k}_{[\beta, \text{भ}]}$ be the *Recognized Precinct-Tally Matrix* **B,**$\boldsymbol{k}_{[\beta, \text{भ}]}$ .

[Linear Algebra — ML Glossary documentation](#)
[Hadamard product (matrices)](#)

The following sheet demonstrates a Precinct Tally Matrix recognizing candidates 1,2,5 in modes 2,4, with an arbitrary criterion that the recorded total for candidate zero in mode zero is less than the recorded total for candidate one in mode one.

$\flat_{\text{अ}} = 1$ for $\chi$ when $\varepsilon = 1, 2, 5$ *and* $\delta = 2, 4$. This can also be written as: $\flat_{\text{अ}} = 1$ for $j$ when $j \equiv 1, 2, 5 \bmod 7$ *and* $\lfloor \frac{j}{7} \rfloor = 2, 4$.
Precinct-Tally Recognition Matrix

From the *Recognized Precinct-Tally Matrix*, we now remove the null (*discarded*) columns and rows and reform the matrix into one of smaller dimensions, $\beta_2 \times \text{अ}_2$, in order to utilize the full power of parallel computation to investigate the current recognized values.

Which now brings us to the final set of definitions for this section: *Reduced, Hybrid, Partition, Focus, Aggregate, Reflection and Zeta.*

## *Definition 1.1.24b; The Reduced Precinct-Tally Matrix*

Let **B,**$\boldsymbol{k}_{[\beta_2, \text{अ}_2]}$ be a $\beta_2 \times \text{अ}_2$ matrix containing only the recognized column and row vectors.

## *Definition 1.1.24c; The Discarded Precinct-Tally Matrix*

Let ¬**B,**$\boldsymbol{k}_{[\neg\beta_2, \neg\text{अ}_2]}$ be the matrix of discarded precincts and $\chi$. This matrix is obtained in the same manner as the *Recognized Precinct-Tally Matrix* by reversing the truth values of $\lozenge_{\beta,z}$ and $\flat_{\text{अ}}$.

The *Recognized Precinct-Tally* and the *Hybrid Matrix* (next definition) are powerful tools to accelerate our computations; however, they cannot be utilized via spreadsheet applications such as *Excel* and *Librecalc*.

*The following definitions of <u>Hybrid</u>, <u>Partition</u>, <u>Focus</u>, <u>Aggregate</u>, <u>Reflection</u> and <u>Zeta</u> shall be defined in a strict mathematical manner that ignores the limitations of spreadsheet applications;however, in order to enable people who only have access and knowledge of spreadsheet applications to replicate these procedures, accompanying spreadsheet examples and detailed instructions of their operationsshall be included. Spreadsheet users shall immediately proceed to the Partition and Focus stages in their computations. It is advised, however, that spreadsheet users read the definitions preceding the <u>Partition</u> and <u>Focus</u> definitions, in order to understand the theory behind their operation.*

*The term <u>Discard</u> is only being used in the computational sense, in reality, the <u>Discarded Precinct-Tally Matrix</u> represents that which was outside the control of the hostile actors who manipulatedthe <u>Recognized Precinct-Tally Matrix</u> to achieve a Town, County, State or Nationwide aggregate result to either force a win for an <u>Intended Winner</u>, such as United States Presidential, United States Congressional Office, State Gubernatorial, County Executive, Coroner or a local school Superintendent position; or, to force a loss for an <u>Intended Loser</u>, such as a Primary or Caucus Event, regardless of who elsewins.*

*The case of an<u>Intended Winner</u> is often easier to solve, since there are usually no more than two candidates that could have realistically won the race, and thus the Candidate and Mode Combinations can usually be deduced by intuition.*

*However, the case of an<u>Intended Loser</u> (perhaps like Bernie Sanders) in a primary or caucus event cannot so be readily deduced, as the algorithm had no intended winner. Rather, the algorithm would only have to diminish or debase the <u>Intended Loser</u>, forcing us to examine every possible combination of modes and candidates.*

*Also, it is possible that the algorithm can fail to produce its intended result when a hard boundary interferes, such as the number of registered voters, or the obvious prohibition of reporting andutilizing negative percentages (or percentages in excess of 100%). The proportion,<u>Zeta</u>, has a maximum bound against the number of registered voters; likewise,both the Aggregate and Reflection percentages have maximum and minimum bound (respectively) against their main components (the Focus and Anti-Focus) of 100% and 0% (respectively).*

*Definition 1.1.25; The Hybrid and Dormant Sets*

Let $\mathbf{H}_{\text{अ} 3}$ be a subset of $\mathbf{B}_{\text{अ} 2}$; $\mathbf{H}_{\text{अ} 3} = \mathbf{B}_{\text{अ}_2, \Delta_2}$**;** this is the *Hybrid Set.* $|\mathbf{H}_{\text{अ} 3}| = \text{अ}_3$.

Let $\neg\mathbf{H}_{\neg\text{अ} 3} = \mathbf{B}_{\text{अ} 2} - \mathbf{H}_{\text{अ} 3}$ be the *Dormant Set*; $\neg\mathbf{H}_{\neg\text{अ} 3} = \mathbf{B}_{\text{अ}_2, \neg\Delta_2}. = \mathbf{B}_{\text{अ}_2, \left(2^{\text{अ}_2} - 1\right) - \Delta_2}$ ; $|\neg\mathbf{H}_{\neg\text{अ} 3}| = \neg\text{अ}_3$.

*Definition 1.1.26; The Partition and Anti-Partition Sets.*

Let $\mathbf{S}_{\text{Ж}}$ be a subset of $\mathbf{H}_{\text{अ} 3}$; $\mathbf{S}_{\text{Ж}} = \mathbf{H}_{\text{अ}_3, \Delta_3}$**;** this is the *Partition Set.* $|\mathbf{S}_{\text{Ж}}| = \text{Ж}$ .

Let $\neg\mathbf{S}_{\neg\text{Ж}} = \mathbf{H}_{\text{अ} 3} - \mathbf{S}_{\text{Ж}}$ be the *Anti-Partition Set*; $\neg\mathbf{S}_{\neg\text{Ж}} = \mathbf{H}_{\text{अ}_3, \neg\Delta_3}. = \mathbf{H}_{\text{अ}_3, \left(2^{\text{अ}_3} - 1\right) - \Delta_3}$**.** $|\neg\mathbf{S}_{\neg\text{Ж}}| = \neg\text{Ж}$ .

*Definition 1.1.27; The Focus and Anti-Focus Sets.*

Let $\mathbf{F}_{\text{Љ}}$ be a subset of $\mathbf{S}_{\text{Ж}}$; $\mathbf{F}_{\text{Љ}} = \mathbf{S}_{\text{Ж}, \Delta_4}$**;** this is the *Focus Set.* $|\mathbf{F}_{\text{Љ}}| = \text{Љ}$ .

Let $\neg\mathbf{F}_{\neg\text{Љ}} = \mathbf{S}_{\text{Ж}} - \mathbf{F}_{\text{Љ}}$ be the *Anti-Focus Set*; $\neg\mathbf{F}_{\neg\text{Љ}} = \mathbf{S}_{\text{Ж}, \neg\Delta_4} = \mathbf{S}_{\text{Ж}, \left(2^{\text{Ж}} - 1\right) - \Delta_4}$ . $|\neg\mathbf{F}_{\neg\text{Љ}}| = \neg\text{Љ}$ .

*Definition 1.1.28; The Contrast and Anti-Contrast Sets.*

Let $\mathbf{G}_{\text{Я}}$ be a subset of $\neg\mathbf{S}_{\text{Ж}}$; $\mathbf{G}_{\text{Я}} = \neg\mathbf{S}_{\text{Ж}, \Delta_5}$**;** this is the *Contrast Set.* $|\mathbf{G}_{\text{Я}}| = \text{Я}$ .

Let $\neg\mathbf{G}_{\neg\text{Я}} = \neg\mathbf{S}_{\text{Ж}} - \mathbf{G}_{\text{Я}}$ be the *Anti-Contrast Set*; $\neg\mathbf{G}_{\neg\text{Я}} = \neg\mathbf{S}_{\text{Ж}, \neg\Delta_5} = \mathbf{S}_{\text{Ж}, \left(2^{\text{Ж}} - 1\right) - \Delta_5}$ . $|\neg\mathbf{G}_{\neg\text{Я}}| = \neg\text{Я}$ .

*Definition 1.1.29; Partition Zeta*

Let $\zeta_{\text{Ж}}$ be proportion of all votes in the *Anti-Partition* to the *Partition*.

$$\zeta_{\text{Ж}} = \frac{\sum_{j=1}^{j=\neg\text{Ж}} \neg s_j}{\sum_{k=1}^{k=\text{Ж}} s_k}$$

*Definition 1.1.30; Focus and Anti-Focus Percentage*

Focus Percentage: $f_{Љ} = \frac{\sum_{t=1}^{t=Љ} f_t}{\sum_{k=1}^{k=Ж} s_k}$ Anti-Focus Percentage: $\neg f_{\neg Љ} = \frac{\sum_{t=1}^{t=Љ} \neg f_t}{\sum_{k=1}^{k=Ж} s_k}$

$1 - f_{Љ} = \neg f_{\neg Љ}$.

*Definition 1.1.31; Contrast and Anti-Contrast Percentage*

Contrast Percentage: $g_{Я} = \dfrac{\sum_{w=1}^{w=Я} g_w}{\sum_{j=1}^{j=\neg Ж} \neg s_j}$ Anti-Contrast Percentage: $\neg g_{\neg Я} = \dfrac{\sum_{w=1}^{w=Я} \neg g_w}{\sum_{j=1}^{j=\neg Ж} \neg s_j}$

$1 - g_{Я} = \neg g_{\neg Я}$.

*Definition 1.1.32; Focus Aggregate and Focus Reflection(Focus to Contrast)*

Focus Aggregate: $\alpha_{Я} = \dfrac{f_{Љ} + (\zeta_{Ж}) g_{Я}}{\zeta_{Ж} + 1}$ Focus Reflection: $\lambda_{Я} = \dfrac{f_{Љ} + (\zeta_{Ж}) \neg g_{\neg Я}}{\zeta_{Ж} + 1}$ $1 - \alpha_{Я} = \lambda_{Я}$.

*Definition 1.1.33; The Focus Aggregate and Reflection to Anti-Contrast Percentage Conversion Identity.*

Conversion: $\neg g_{\neg Я} = \dfrac{\lambda_{Я}(\zeta_{Ж}+1) + (\zeta_{Ж}) - \alpha_{Я}(\zeta_{Ж}+1)}{2\zeta_{Ж}}$

## *Theorems 1.1.34; The Hyperbolic Theorems, Proofs by Reflexivity*

Let $\mathbf{A_1} \cap \mathbf{A_2} = \varnothing$.
Let $\mathbf{A_3} \cap \mathbf{A_4} = \varnothing$.
Let $\mathbf{A_1} \cup \mathbf{A_2} = \mathbf{X}$
Let $\mathbf{A_3} \cup \mathbf{A_4} = \mathbf{Y}$
Let $\mathbf{X} \cap \mathbf{Y} = \varnothing$.
Let $\mathbf{X} \cup \mathbf{Y} = \mathbf{\Omega}$.
Let $\mathbf{A_1} \cup \mathbf{A_3} =$
Let $|\mathbf{A_1}| = a$.
Let $|\mathbf{A_2}| = b$.
Let $|\mathbf{A_3}| = c$.
Let $|\mathbf{A_4}| = d$.
Let $|\mathbf{X}| = x = a + b$.
Let $|\mathbf{Y}| = y = c + d$.
Let $|\mathbf{\Omega}| = \omega = x + y$.
Let $| \ | = k = a + c$.
Let $\zeta = \frac{y}{x} = \frac{c+d}{a+b}$

Then let:

$$\alpha = \frac{a+c}{\omega} = \frac{k}{\omega} = \frac{\frac{a}{x}+\zeta\frac{c}{y}}{\zeta+1} \quad ; \quad \lambda = \frac{a+d}{\omega} = \frac{\frac{a}{x}+\zeta\frac{d}{y}}{\zeta+1}$$

[Formal logic - Logical manipulations in LPC]
[Reflexive relation]
[The Reflexive Property of Equality: Definition & Examples - Video & Lesson Transcript]

## *Lemma 1.1.34a; The Twixt Lemma*

The proportion of the elements in two disjoint subsets, $\mathbf{A_1}$ and $\mathbf{A_3}$, of the same superset, $\mathbf{\Omega}$, to the elements in $\mathbf{\Omega}$, always exists between the proportion of the elements in $\mathbf{A_1}$ to the elements in the set $\mathbf{X} = \mathbf{A_1} \cup \mathbf{A_2}$, $\mathbf{A_1} \cap \mathbf{A_2} = \varnothing$, $\mathbf{A_2} \subset \mathbf{\Omega}$ , and the proportion of the elements in $\mathbf{A_3}$ to the elements in the set $\mathbf{Y} = \mathbf{A_3} \cup \mathbf{A_4}$, $\mathbf{A_3} \cap \mathbf{A_4} = \varnothing$, $\mathbf{A_4} \subset \mathbf{\Omega}$ , if, and only if: $\bigcap_{i=1}^{i=4} \mathbf{A_i} = \varnothing$ and $\bigcup_{i=1}^{i=4} \mathbf{A_i} = \mathbf{\Omega}$.

Thus, the aggregate proportion $\frac{|A_1|+|A_3|}{|A_1|+|A_2|+|A_3|+|A_4|}$ is always bounded between $\frac{|A_1|}{|A_1|+|A_2|}$ and $\frac{|A_3|}{|A_3|+|A_4|}$

***Proof*:**
Given $\frac{a}{a+b} \le \frac{c}{c+d}$, then show: $\frac{a}{a+b} \le \frac{a+c}{a+b+c+d} \le \frac{c}{c+d}$

1. $\frac{a}{a+b} \le \frac{c}{c+d} \rightarrow ac + ad \le ac + cb \rightarrow ad \le bc$

2. $\frac{a}{a+b} \le \frac{a+c}{a+b+c+d} \rightarrow a^2 + ab + ac + ad \le a^2 + ac + ba + bc \rightarrow ad \le bc$

3. $\frac{a+c}{a+b+c+d} \le \frac{c}{c+d} \rightarrow ac + ad + c^2 + cd \le a^2 + ac + ba + bc \rightarrow ad \le bc$

Q.E.D

*Lemma 1.1.34b; The Aggregate Lemma*

Aggregate Lemma: $\frac{k}{\omega} = \frac{\frac{a}{x}+\zeta\frac{c}{y}}{\zeta+1}$

Given: $\bigcap_{i=1}^{i=4} \mathbf{A_i} = \varnothing$ and $\bigcup_{i=1}^{i=4} \mathbf{A_i} = \mathbf{\Omega}$.

Then the proportion of the elements in to the elements in $\mathbf{\Omega}$ is equal to the proportion of the areas:

1. The combined area of a rectangle of height 1 and width $\frac{a}{x}$ and the area of a rectangle of height $\xi$ and width $\frac{c}{y}$.
2. The area of rectangle of height 1 and a width of $\zeta + 1$.

   Whereas the proportion of the elements in to $\mathbf{\Omega}$ is a rectangle of height 1 and width $\frac{k}{\omega}$ to a square of side length 1, such that the proportion of the area between two squares of side lengths…

3. $\sqrt{\frac{k}{\omega}}$ and $\sqrt{1}$

   ... is equal to the proportion of the two squares of side lengths…

4. $\sqrt{\frac{a}{x} + \zeta\frac{c}{y}}$ to $\sqrt{\zeta + 1}$

   and therefore:

5. $\sqrt{\frac{k}{\omega}}$ to $1$ and $\frac{\sqrt{\frac{a}{x}+\zeta\frac{c}{y}}}{\sqrt{\zeta+1}}$ to $1$ implies that $\sqrt{\frac{k}{\omega}} = \frac{\sqrt{\frac{a}{x}+\zeta\frac{c}{y}}}{\sqrt{\zeta+1}}$ and thus: $\frac{k}{\omega} = \frac{\frac{a}{x}+\zeta\frac{c}{y}}{\zeta+1}$

***Proof*:**

$$\frac{k}{\omega} = \frac{\frac{a}{x}+\zeta\frac{c}{y}}{\zeta+1}$$

$$\frac{a+c}{a+b+c+d} = \frac{\left(\frac{a}{a+b}\right)+\left(\frac{c+d}{a+b}\right)\left(\frac{c}{c+d}\right)}{\left(\frac{c+d}{a+b}\right)+1}$$

$$\left(\left(\frac{c+d}{a+b}\right) + 1\right)\left(\frac{a+c}{a+b+c+d}\right) = \left(\frac{a}{a+b}\right) + \left(\frac{c}{a+b}\right)$$

$$\left(\frac{c+d+a+b}{a+b}\right)\left(\frac{a+c}{a+b+c+d}\right) = \left(\frac{1}{a+b}\right)(a) + \left(\frac{1}{a+b}\right)(c)$$

$$\left(\frac{1}{a+b}\right)\left(\frac{c+d+a+b}{1}\right)\left(\frac{1}{a+b+c+d}\right)\left(\frac{a+c}{1}\right) = \left(\frac{1}{a+b}\right)(a + c)$$

$$a + c = a + c$$

Q.E.D

*Theorem 1.1.34c; The Hyperbolic Reflection Theorem*

Hyperbolic Reflection Theorem: $\frac{1}{\zeta_i} = \frac{a_{i,j} - r_{i,j,2}}{r_{i,j,1} - a_{i,j}}$

Given: $\bigcap_{i=1}^{i=4} \mathbf{A}_i = \varnothing$ and $\bigcup_{i=1}^{i=4} \mathbf{A}_i = \mathbf{\Omega}$.

Let the difference between the proportions of...

1. The proportion of the elements in two disjoint subsets, $\mathbf{A}_1$ and $\mathbf{A}_3$, of the same superset, $\mathbf{\Omega}$, to the elements in $\mathbf{\Omega}$;
2. The proportion of the elements in $\mathbf{A}_3$ to the elements in the set $\mathbf{Y} = \mathbf{A}_3 \cup \mathbf{A}_4$;

… be equal to $d_2$.

$$d_2 = \frac{k}{\omega} - \frac{c}{y} = \frac{a+c}{a+b+c+d} - \frac{c}{c+d}$$

Let the difference between the proportions of...

3. The proportion of the elements in $\mathbf{A}_1$ to the elements in the set $\mathbf{X} = \mathbf{A}_1 \cup \mathbf{A}_2$;
4. The proportion of the elements in two disjoint subsets, $\mathbf{A}_1$ and $\mathbf{A}_3$, of the same superset, $\mathbf{\Omega}$, to the elements in $\mathbf{\Omega}$;

… be equal to $d_1$.

$$d_1 = \frac{a}{x} - \frac{k}{\omega} = \frac{a}{a+b} - \frac{a+c}{a+b+c+d}$$

Then

$$\frac{d_2}{d_1} = \frac{1}{\zeta} = \frac{x}{y} = \frac{a+b}{c+d} \rightarrow \zeta = \frac{c+d}{a+b} = \frac{\left(\frac{a}{x} - \frac{k}{\omega}\right)}{\left(\frac{k}{\omega} - \frac{c}{y}\right)} = \frac{\left(\frac{a}{a+b} - \frac{a+c}{a+b+c+d}\right)}{\left(\frac{a+c}{a+b+c+d} - \frac{c}{c+d}\right)}$$

Thus any particular candidate's percentage of the vote in the second mode (where the candidate's percentage is equal to the height $r_{i,j,2}$) is a vertical reflection of the candidate's percentage of the vote in the first mode ($r_{i,j,1}$) over the candidate's aggregate percentage of both modes (height is equal to $a_{i,j}$), such that the distance from $r_{i,j,2}$ to $a_{i,j}$ is negative inversely proportional to the distance from $r_{i,j,1}$ to $a_{i,j}$ in respect to ζ, which is the proportion of the total votes in mode two to mode one. $\frac{1}{\zeta_i} = \frac{-r_{i,j,2} + a_{i,j}}{r_{i,j,1} - a_{i,j}} = \frac{a_{i,j} - r_{i,j,2}}{r_{i,j,1} - a_{i,j}}$

***Proof***:

$$\left(\frac{c+d}{a+b}\right) = \frac{\left(\frac{a}{a+b} - \frac{a+c}{a+b+c+d}\right)}{\left(\frac{a+c}{a+b+c+d} - \frac{c}{c+d}\right)}$$

$$\left(\frac{a+c}{a+b+c+d} - \frac{c}{c+d}\right)\left(\frac{c+d}{a+b}\right) = \left(\frac{a}{a+b} - \frac{a+c}{a+b+c+d}\right)$$

$$\left(\frac{ac+ad+c^2+cd-ac-bc-c^2-cd}{(a+b+c+d)(c+d)}\right)\left(\frac{1}{a+b}\right)(c\ +\ d) = \left(\frac{a^2+ab+ac+ad-a^2-ac-ab-bc}{(a+b+c+d)(a+b)}\right)$$

$$\left(\frac{c+d}{1}\right)\left(\frac{1}{c+d}\right)\left(\frac{ac+ad+c^2+cd-ac-bc-c^2-cd}{(a+b+c+d)}\right)\left(\frac{1}{a+b}\right)=\left(\frac{1}{a+b}\right)\left(\frac{a^2+ab+ac+ad-a^2-ac-ab-bc}{(a+b+c+d)}\right)$$

$$ac+ad+c^2+cd-ac-bc-c^2-cd=a^2+ab+ac+ad-a^2-ac-ab-bc$$

$$ad-bc=ad-bc$$

Q.E.D

*Theorem 1.1.34d; Aggregate-Reflection Conversion Theorem:*

$$(r_{i,3-j,2})=\frac{\lambda_{i,j}(\zeta_i+1)+\zeta_i-a_{i,1}(\zeta_i+1)}{2\zeta_i};\ \frac{d}{c+d}=\frac{\lambda(\zeta+1)+\zeta-\alpha(\zeta+1)}{2\zeta}$$

Given: $\bigcap_{i=1}^{i=4}\mathbf{A}_i=\varnothing$ and $\bigcup_{i=1}^{i=4}\mathbf{A}_i=\mathbf{\Omega}$; $\lambda=\frac{\left(\frac{a}{a+b}\right)+\left(\frac{c+d}{a+b}\right)\left(\frac{d}{c+d}\right)}{\left(\frac{c+d}{a+b}+1\right)}$; $\alpha=\frac{\left(\frac{a}{a+b}\right)+\left(\frac{c+d}{a+b}\right)\left(\frac{c}{c+d}\right)}{\left(\frac{c+d}{a+b}+1\right)}$; $\mu=\frac{d}{c+d}=\frac{d}{y}$

$$2\zeta\mu=\zeta+(\lambda(\zeta+1)-\alpha(\zeta+1))$$

The area of a rectangle of height $\frac{c+d}{a+b}$ and a width of $(1)$, combined with the difference between between the areas of two rectangles of heights $\left(\frac{c+d}{a+b}+1\right)$ and widths $\frac{a+d}{a+b+c+d}$ and $\frac{a+c}{a+b+c+d}$, is equal to the area of a rectangle of height $2\,\frac{c+d}{a+b}$ and width $\frac{d}{c+d}$.

***Proof***:

$$(2)\left(\frac{c+d}{a+b}\right)\left(\frac{d}{c+d}\right)=\left(\frac{c+d}{a+b}\right)+\left(\frac{a+d}{a+b+c+d}\right)\left(\frac{c+d}{a+b}+1\right)-\left(\frac{a+c}{a+b+c+d}\right)\left(\frac{c+d}{a+b}+1\right)$$

$$(2)\left(\frac{c+d}{a+b}\right)\left(\frac{d}{c+d}\right)=\left(\frac{c+d}{a+b}\right)+\left(\frac{c+d}{a+b}+1\right)\left(\frac{a+d}{a+b+c+d}-\frac{a+c}{a+b+c+d}\right)$$

$$(2)\left(\frac{c+d}{a+b}\right)\left(\frac{d}{c+d}\right)=\left(\frac{c+d}{a+b}\right)+\left(\frac{c+d}{a+b}+1\right)\left(\frac{a+d-a-c}{a+b+c+d}\right)$$

$$(2)\left(\frac{d}{a+b}\right)=\left(\frac{c+d}{a+b}\right)+\left(\frac{a+b+c+d}{a+b}\right)\left(\frac{d-c}{a+b+c+d}\right)$$

$$(2)\left(\frac{d}{a+b}\right)=\left(\frac{c+d}{a+b}\right)+\left(\frac{d-c}{a+b}\right)$$

$$\frac{2d}{a+b}=\frac{2d}{a+b}$$

Q.E.D

### *Commentary 1.1.35; Reflection Fraud*

The above theorem alerts us to the fact that malevolent actors can achieve a predetermined aggregate, $\alpha$, for a precinct with only the following values:

1. The first candidate's aggregate percentage, $\alpha$.
2. The abstract aggregate (reflection) of those who voted for the first candidate in the first mode OR who voted for the second candidate in the second mode, $\lambda$.
3. The proportion of all who voted in the second mode against all who voted in the first mode, $\zeta$.

From those three values, the second candidate's percentage in the second mode is immediately derived, $\mu = \frac{d}{c+d}$, forcing the first candidate's percentage in the second mode to materialize, $1 - \mu = 100\% - \mu$; and forcing the first candidate's percentage in the first mode to materialize, as $\lambda = \frac{a+d}{\omega} = \frac{\frac{a}{x}+\zeta\frac{d}{y}}{\zeta+1}$ implies $\frac{a}{x} = \frac{a}{a+b} = (\zeta + 1)\lambda - \zeta\frac{d}{c+d} = \lambda\zeta + \lambda - \zeta\frac{d}{c+d} = \lambda + \zeta\left(\lambda - \frac{d}{c+d}\right)$, and therefore the second candidate's percentage in the first mode also comes into being, since: $\frac{b}{a+b} = 1 - \frac{a}{a+b}$.

This chain reaction of conversions can be utilized to prevent a candidate from falling below a 50% aggregate, regardless of their performance in the first mode, namely the critical case of $\zeta = 3, \lambda = 25\%, 0\% \leq \frac{a}{a+b} \leq 100\%$... forcing $\frac{a+c}{a+b+cd} = \alpha \geq 50\%$, with all $\frac{b}{a+b}, \frac{c}{c+d}, \frac{d}{c+d}$ also bounded between 0% and 100%.

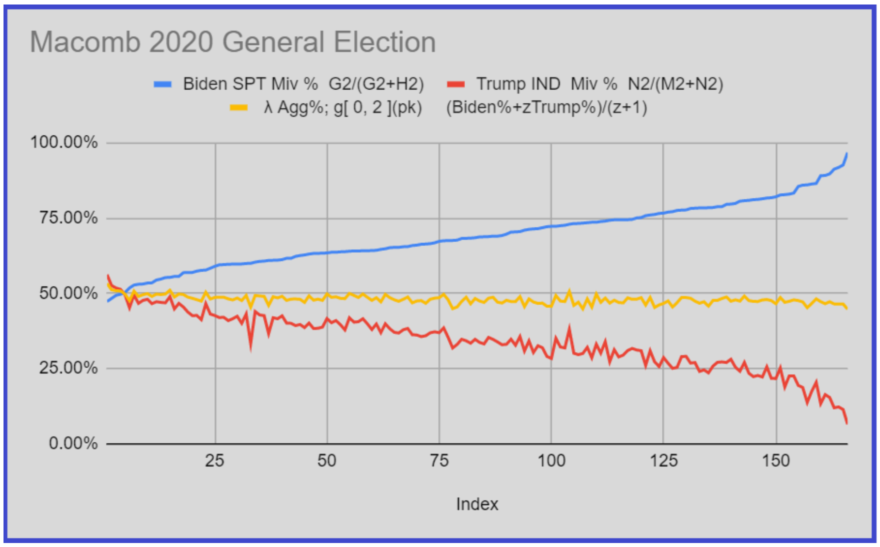
https://www.desmos.com/calculator/kgukbggloh

*Figure 1.1.35; Michigan, The Monster of Macomb*

Although such a scenario would seem preposterous, let us consider the following data set from the Macomb County Recorder in the State of Michigan, in the United States 2020 Presidential Election, after the third party candidates are removed from the Precinct Tally Matrix (discarded): Macomb Reflection

Macomb Co. - Election Results

*Precinct Criterion* Ⅰ : Trump's Combined Election Day and Mail-In Straight Party Ticket Percentage **less than** his combined Election Day and Individual Percentage.
**167 Precincts**, *x*-axis.

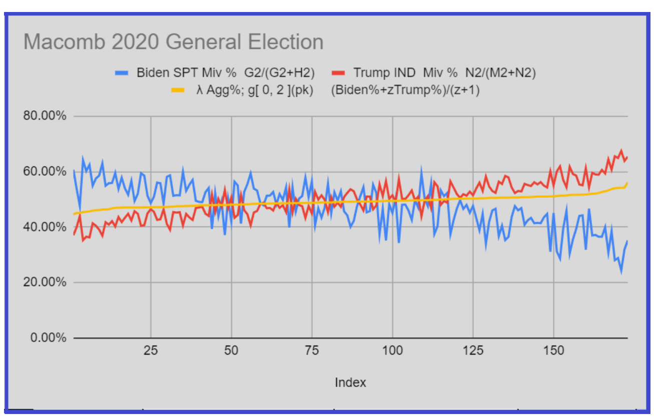



It turns out that in every county where wrong-doing was alleged, that the reflection constant, λ , was held virtually … constant!

The question is: No matter how obvious and ridiculous it would be to propose that the aggregate percentage of people who voted for Biden Straight Party Ticket in the mail and the people who voted for Trump individually in the mail always came out to exactly 48% of the entire mail-in vote, with virtually no variance, was a natural event in a safe and secure election, how do we prove otherwise that this result was manufactured, and not just a freak act of nature that struck each largely populated county in Michigan?

Thankfully the tests are quite simple; however, the theory behind these tests must first pass peer review, as such, this is the entire purpose of this article!

Precinct Criterion Ⅱ: Trump's Combined Election Day and Mail-In Straight Party Ticket Percentage **greater than** his combined Election Day and Individual Percentage. All remaining 173 Precincts, *x*-axis. λ Average of 49.27%; standard deviation of 1.95%.



# Section Ⅱ, Precinct Orderings and Rankings and Election Functions

### *Definition 1.2.1; Alpha-Numeric Ordering*

Let $\hbar_0(p_i)$ be the function that returns the alphanumeric ranking of a precinct, such that if the precincts were ordered from least to greatest by standard alphanumeric procedure , then $\hbar_0(p_i)$ would be the position of that precinct in a list of all the precincts sorted alphanumerically.

$\hbar_0(p_i)$ is the function to which our Precinct Index shall be set to. This method is preferable to sorting by two more or columns of town, ward and precinct names.

### *Definition 1.2.2; Rank Function*

Let $f(p_i)$ be a function of a set of precincts,then:

Let $(\hbar \circ f)(p_i)$ returns the rank of that precinct in respect to the function $f(x)$, such that if the precincts were ordered from least to greatest by their evaluations of $f(p_i)$, then $(\hbar \circ f)(p_i)$ would be the position of that precinct in a list of all the precincts sorted from least to greatest by their evaluations of $f(p_i)$.

If there exists a tie in the rankings, such that $f(p_i)=f(p_j)$ then the alphanumeric function $\hbar_0(p)$ shall break the tie between those precincts, such that...

If $\hbar_0(p_i) < \hbar_0(p_j)$, then $\hbar_f(p_i) < \hbar_f(p_j)$ (and vice versa)...

...ensuring that there exists a *bijection* between the set of precincts and the ranks of the precincts for any function performed upon them.

For spreadsheet applications, such as Excel or Librecalc, we use the following function instead to break the ties:

=RANK(A2,$A$2:$A$340,1)+COUNTIF($A$2:A2,A2)-1

The above spreadsheet function will decide any ties and maintain a bijection between the 339 rows (in the code) and the ranks.
We can check this sum with $\sum_{n=1}^{k=n} k = \frac{1}{2}\left(n^2 + n\right)$, where *n* is the number of rows bounding the data (column) being ranked.

### *Definition 1.2.3; Quantile Indexing Function*

Let β be the number of entities measured by $f(x)$, then let $(q \circ \hbar \circ f)(p_i) = \frac{(\hbar \circ f)(p_i)}{\beta}$ be the quantile indexing function.

In this article all arbitrary regression functions shall be executed against the quantiles from $\frac{1}{\beta}$ to $\frac{\beta}{\beta}$ instead of their integer indices. Thus, when $f(x)$ is returning a percentage of candidate or aggregate, then both the *x*-axis (the quantiles) and the *y*-axis (the percentage measured) shall be bound strictly within the unit square, allowing us to take the area under the curve of the regression lines to determine the candidate's performance across the entire county.

### *Definition 1.2.4; Quantile Domain Function*

Let $f_q(x)$ be the horizontal compression of $f(x)$, such that the original domain$\{x|\ 1 \leq x \leq \beta\},\ x \in \mathbb{Z}$ is compressed to $\left\{x|\ \frac{1}{\beta} \leq x \leq 1\right\},\ x \in \mathbb{Q}$.

### *Definition 1.2.5; Quantile Range Function*

Let $\left(\frac{1}{h_1}\right)f_q\ (x)$ be the vertical compression of $f_q(x)$, such that the absolute value original range$\left\{y|\ 0 \leq |y| \leq h_1\right\},\ y \in \mathbb{R})$ is compressed between 0 and 1, restricting the domain and range of $\left(\frac{1}{h_1}\right)f_q\ (x)$to the unit square.

### *Definition 1.2.6; The Quantile Range Midpoint Constants*

Let $t_1$and $t_2$ be the minimum and maximum of $f_q(x)$, then let $m_2 = \frac{1}{2}\left(t_1 + t_2\right)$ and let $h_3 = \frac{1}{2}\left(t_2 - t_1\right)$

### *Definition 1.2.6; Quantile Origin Function*

Let $-\ h_2 + \left(\frac{1}{h_3}\right)f_q\ (x + \frac{1}{2})$ be the horizontal and vertical translation of the data set from the unit square to the unit circle. If any outliers are removed, it must be specified and be made known to have been removed from the data set.

### *Definition 1.2.7; Quantile Projection Function*

If $y \in \mathbb{C}$, then let P:$f_q(x)$ be the orthogonal projection of $y = a + bi$ to $y = a$

### *Definition 1.2.7; Quantile Magnitude Function*

If $y \in \mathbb{C}$, then let M:$f_q(x)$ be the conversion of $y = f_q(x) = a + bi$ to $y = \sqrt{a^2 + b^2}$.

### *Definition 1.2.4; Regression Function*

Let $(g \circ q \circ \text{ƻ} \circ f\,)(p_i)$ be a regression function of $f(x)$ against the quantiles.

For convenience, we shall simply denote $(g \circ q \circ \text{ƻ} \circ f\,)(p_i)$ as $(g \circ f)(p_i)$, so long as the context implies the ordering of the precincts.

### *Definition 1.2.5; Residual Function*

Let $(\bowtie \circ\, g \circ q \circ \text{ƻ} \circ f\,)(p_i)$the residual function: $\bowtie(p_i) = (g \circ q \circ \text{ƻ} \circ f\,)(p_i) - g(p_i)$.

Example:

Let $f_{r_{j,2}}(p_i)$ be the function that returns Fiona's Mail-in Percentages, then (based on previous tables):

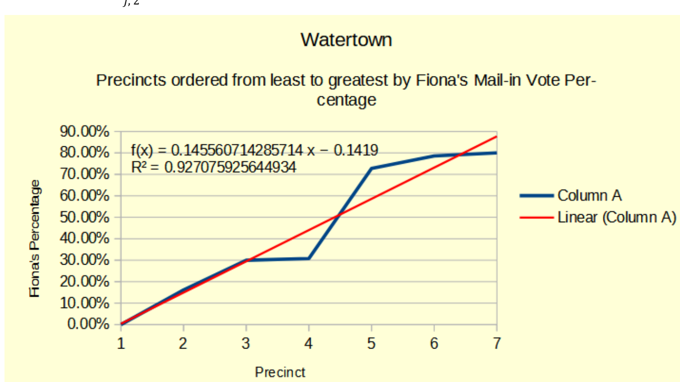
Alphanumeric Ordering

| Precinct | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ | $p_7$ |
|---|---|---|---|---|---|---|---|
| $h_0(p_i)$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $f_{r_{j,2}}(p_i)$ | 30.00% | 72.73% | 78.58% | 30.77% | 00.00% | 16.16% | 80.00% |
| $\left(h \circ f_{r_{j,2}}\right)(p_i)$ | 3 | 5 | 6 | 4 | 1 | 2 | 7 |

$\left(h \circ f_{r_{j,2}}\right)(p_i)$ Ordering (Fiona's Mail-in Ordering):

| Precinct | $p_5$ | $p_6$ | $p_1$ | $p_4$ | $p_2$ | $p_3$ | $p_7$ |
|---|---|---|---|---|---|---|---|
| $f_{r_{j,2}}(p_i)$ | 00.00% | 16.16% | 30.00% | 30.77% | 72.73% | 78.58% | 80.00% |
| $g_{r_{j,2}}(p_i)$ | 00.37% | 14.92% | 29.48% | 44.03% | 58.59% | 73.15% | 87.70% |
| $\varepsilon(p_i)$ | -0.37% | 1.24% | 0.52% | -13.26% | 14.14% | 5.43% | -7.70% |

$$g_{r_{j,2}}(p_i) = -0.1419 + 0.1455((h \circ f)(p_i))$$



# Section Ⅲ, Predetermined Aggregate Against a Natural Mode

In this section we shall learn how to detect when the second mode is adjusted to achieve a predetermined aggregate against the first mode.

### *Resource 1.3.1; Pre-Determined Aggregate Simulator, Mail-in Adjustment*

Pre-Determined Aggregate Generator

See Appendix A for details.

[Fair Election Generator]
[Pre-Determined Aggregate Generator, EDV Adjustment]
*Sfsdfs*

*[Macomb SPT vs Indv Partition 1 (desmos.com)]*

*https://www.desmos.com/calculator/lba2g2j7k3*

## *Definition 1.3.2; More Stuff*

Let **C** be the set of candidates, such that $c_1$ is the first candidate and $c_2$ is the second candidate in an election. For this chapter, the first candidate will be named *John* and the second candidate will be named *Fiona*.

Reflection Fraud

LHS is secant fraud RHS Differentiation Fraud

https://www.desmos.com/calculator/ad3i6e5j5a

# Appendix A, Pre-Determined Aggregate Generator

This generator looks only at a single candidate in terms of percentages only. Mode 1 of voting is theElection Day Vote, Mode 2 of voting is the Mail-in Vote. The aggregate is the combination of both votes in respect to zeta.

The Generator begins in *Column G*. This column generates the election day vote with a set mean and standard deviation, NORM.INV(RAND(),$AG$3,$AG$4), where cells *AG3* and *AG4* are the mean and standard deviation.

In *Column H* the Predetermined Aggregate is generated in the same manner: =NORM.INV(RAND(),$AG$7,$AG$8)

In *Column B* the Election Day Vote is sorted from least to greatest, =SMALL($G$2:$G$1001,A2), and *Column A* indexes this order. Thus *Column B* = $f_{r_{i,j,1}}(p_i)$ and *Column A* = $\left(h \circ f_{r_{j,2}}\right)(p_i)$.

In *Column F* the Aggregate Vote is sorted from least to greatest, =SMALL($H$2:$H$1001,A2), thus thus *Column B* = $f_{r_{i,j,aggregate}}(p_i)$ .

In *Column C* we add normal noise to the aggregate vote seen in *Column F* to ensure that a correlation exists between the Election Day Vote and the Aggregate Vote, whilst also ensuring that the precincts, when sorted from least to greatest by their aggregate percentage, do not remain in the exact same order as Election Day Sorting. =F2+NORM.INV(RAND(),$AG$11,$AG$12).

In *Column E* the Zeta is generated with a set mean and standard deviation: =NORM.INV(RAND(),$AG$15,$AG$16)

In *Column D* the Mail-in Vote is then calculated via *Identity 2* (Lemma 1.1.10): =C2-(B2-C2)/E2. Thus *Column B* = $f_{r_{i,j,2}}(p_i)$ and *Column A* = $\left(h \circ f_{r_{j,2}}\right)(p_i)$.

In *Column I* is the cubic regression of the Mail-in Vote (in the election day ordering). For this experiment cubic regression suffices for normally distributed vote percentages, since odd degree polynomials are the Maclaurin approximations for the Inverse Error Function (quantile function). Inverse Erf -- from Wolfram MathWorld. The 4x4 Design Matrix of the cubic regression is found in cells *AB2:AE5*, whose values were simplified with power summations. Power Summations, as the values of $x$ are a sequence of consecutive integers.

The inverse of this matrix is then calculated and multiplied against the column vector (array):

$r_t = \sum_{i=1}^{i=\beta} x_i^t y_i;\ 0 \le t \le 3$, this vector is present in cells *AF2:AF5*.

The product of inverse matrix with this column vectors returns the coefficients of the regression $\left(a_0,\ a_1,\ a_2, a_3\right)$.

Least Squares Fitting--Polynomial -- from Wolfram MathWorld
Polynomial Regression Model: Derivation: Part 1 of 2
Polynomial Regression Model: Derivation: Part 2 of 2

Thus *Column I* = $\left(g_{r_{i,j,2}} \circ h \circ f_{r_{i,j,1}}\right)(p_i).\varepsilon(p_i) = f(p_i) - g(p_i)$.

In *Column J* we subtract the average value of zeta from each value of zeta, since the values of zetaare normally distributed, bear in mind that such a subtraction is not necessary for this experiment, but is simply a convention to draw the values of zeta towards to the origin.

In *Column K* we determine the residual of the Mail-in Vote against the regression of the Mail-in Vote; thus *Column I* = $\varepsilon(p_i) = f_{r_{i,j,1}}(p_i) - \left(g_{r_{i,j,2}} \circ h \circ f_{r_{i,j,1}}\right)(p_i)$, from which we then draw the scatter plot of these residuals against zeta, where the axes of the scatter plot are $x$ = *Column J and* $y$ = *Column I.*